XCadabra: examples/bianchi_identities.cnb

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$$-\frac{1}{6}\nabla_i\nabla_i(C_{jmnk}C_{mpqn}C_{pjkq}+\frac{1}{2}C_{jkmn}C_{pqmn}C_{jkpq});$$

First apply the product rule to write out the derivatives,

```
@distribute!(%): @prodrule!(%):
@distribute!(%): @prodrule!(%):

@prodsort!(%): @canonicalise!(%): @rename_dummies!(%):
@collect_terms!(%);
```

$$\begin{aligned}
exp := {} & C_{ijmn}C_{ikmp}\nabla_q\nabla_j C_{nkpq} - C_{ijmn}\nabla_k C_{ipmq}\nabla_p C_{jqnk} - 2\,C_{ijmn}\nabla_i C_{mkpq}\nabla_p C_{jknq} \\
& - C_{ijmn}\nabla_k C_{ikmp}\nabla_q C_{jpnq} + C_{ijmn}C_{ikmp}\nabla_j\nabla_q C_{nkpq} - 2\,C_{ijmn}\nabla_i C_{jkmp}\nabla_q C_{nkpq} \\
& - C_{ijmn}C_{ikpq}\nabla_m\nabla_p C_{jqnk} - \frac{1}{4}\,C_{ijmn}C_{ijkp}\nabla_q\nabla_m C_{nqkp} + \frac{1}{4}\,C_{ijmn}\nabla_k C_{ijpq}\nabla_p C_{mnkq} \\
& - \frac{1}{2}\,C_{ijmn}\nabla_i C_{jkpq}\nabla_k C_{mnpq} - \frac{1}{4}\,C_{ijmn}\nabla_k C_{ijkp}\nabla_q C_{mnpq} - \frac{1}{4}\,C_{ijmn}C_{ijkp}\nabla_m\nabla_q C_{nqkp} \\
& - \frac{1}{2}\,C_{ijmn}\nabla_i C_{mnkp}\nabla_q C_{jqkp} + \frac{1}{4}\,C_{ijmn}C_{ikpq}\nabla_j\nabla_k C_{mnpq} \\
& - \frac{1}{2}\,C_{ijmn}C_{ijmk}\nabla_p\nabla_q C_{npkq} + C_{ijmn}\nabla_k C_{ijmp}\nabla_q C_{nqkp} - C_{ijmn}\nabla_k C_{ijmp}\nabla_q C_{nkpq} \\
& + \frac{1}{2}\,C_{ijmn}C_{ikpq}\nabla_m\nabla_j C_{nkpq} + \frac{1}{2}\,C_{ijmn}\nabla_i C_{mkpq}\nabla_n C_{jkpq} - \frac{1}{2}\,C_{ijmn}\nabla_i C_{jkpq}\nabla_m C_{nkpq} \\
& + \frac{1}{2}\,C_{ijmn}C_{ikpq}\nabla_j\nabla_m C_{nkpq} + \frac{1}{2}\,C_{ijmn}C_{ikmp}\nabla_q\nabla_q C_{jknp} + C_{ijmn}\nabla_k C_{ipmq}\nabla_k C_{jpnq} \\
& - \frac{1}{4}\,C_{ijmn}C_{ijkp}\nabla_q\nabla_q C_{mnkp} - \frac{1}{2}\,C_{ijmn}\nabla_k C_{ijpq}\nabla_k C_{mnpq};
\end{aligned}$$

Because the identity which we intend to prove is only supposed to hold on Einstein spaces, we set the divergence of the Weyl tensor to zero,

```
@substitute!(%)( \nabla_{i}{C_{k i l m}} -> 0, \nabla_{i}{C_{k m l i}} -> 0 );
```

$$\begin{aligned}
exp := {} & C_{ijmn}C_{ikmp}\nabla_q\nabla_j C_{nkpq} - C_{ijmn}\nabla_k C_{ipmq}\nabla_p C_{jqnk} - 2\,C_{ijmn}\nabla_i C_{mkpq}\nabla_p C_{jknq} \\
& - C_{ijmn}C_{ikpq}\nabla_m\nabla_p C_{jqnk} - \frac{1}{4}\,C_{ijmn}C_{ijkp}\nabla_q\nabla_m C_{nqkp} + \frac{1}{4}\,C_{ijmn}\nabla_k C_{ijpq}\nabla_p C_{mnkq} \\
& - \frac{1}{2}\,C_{ijmn}\nabla_i C_{jkpq}\nabla_k C_{mnpq} + \frac{1}{4}\,C_{ijmn}C_{ikpq}\nabla_j\nabla_k C_{mnpq} \\
& + \frac{1}{2}\,C_{ijmn}C_{ikpq}\nabla_m\nabla_j C_{nkpq} + \frac{1}{2}\,C_{ijmn}\nabla_i C_{mkpq}\nabla_n C_{jkpq} - \frac{1}{2}\,C_{ijmn}\nabla_i C_{jkpq}\nabla_m C_{nkpq} \\
& + \frac{1}{2}\,C_{ijmn}C_{ikpq}\nabla_j\nabla_m C_{nkpq} + \frac{1}{2}\,C_{ijmn}C_{ikmp}\nabla_q\nabla_q C_{jknp} + C_{ijmn}\nabla_k C_{ipmq}\nabla_k C_{jpnq} \\
& - \frac{1}{4}\,C_{ijmn}C_{ijkp}\nabla_q\nabla_q C_{mnkp} - \frac{1}{2}\,C_{ijmn}\nabla_k C_{ijpq}\nabla_k C_{mnpq};
\end{aligned}$$

This expression should vanish upon use of the Bianchi identity. By expanding all tensors using their Young projectors, this becomes manifest.